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Mathematics Teachers and Their Understanding of Multiplication: A Case of Two African Language-speakers in South Africa

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ABSTRACT This paper reports on data collected from African language-speakers in South Africa whose experience of mathematics teaching takes place in a language that is not their own. The work is theoretically underpinned in the social and situated perspectives of mathematics learning. Major findings from the case of two African language-speakers reported here provide a strong argument for the consideration of the role and structure of language in the teaching and learning of multiplication to African speaking learners in the multilingual classroom. The importance of paying attention to the difference that diverse language speakers attach meaning to multiplied numbers is highlighted in order to ensure that their representation in not lost in the translation. The paper also suggest alternative ways of expressing multiplied numbers in African languages in ways that will not compromise their understanding of this operation, particularly those involving algebraic terms.

INTRODUCTION

The research reported in this paper was conceived by a conversation between the first author and a Grade 11 African language-speaking learner of Mathematics Literacy who said that he had not understood $3x$ to mean three x's at the time he was doing mathematics at the lower levels of his schooling. He told the first author that he believed he would have performed better in Mathematics had he had such kind of understanding of algebraic terms. Taking into account that many learners in the South African education system abandon Mathematics when they reach the Further Education and Training (FET) phase (that is Grades 10 to 12), the matter calls for further investigation. All learners, at the end of Grade 9 (Senior phase), have to select whether they will take Mathematics or Mathematical Literacy in their FET studies. It is a commonly shared view amongst learners and some teachers indicate that Mathematical Literacy is an easier option in terms of studying than Mathematics, but success in the former does not provide access to some university studies and careers (Julie 2006).

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The year 2014 marked eight years since the subject of Mathematical Literacy was introduced in the South African system. It is relevant to know that during this period, Mathematical Literacy has been more presented in "schools for the poor" where the African languages are dominant, in comparison to the most frequent choice of Mathematics in "schools for the rich" where English and/or Afrikaans are dominant (Adler and Pillay, in press). South Africa (SA) has eleven official languages. Besides English and Afrikaans, the other languages are IsiZulu, IsiXhosa, IsiNdebele, IsiSwati, SeSotho, SeTswana, SePedi, TsiVhenda and XiTsonga, which are spoken by mainly by the black population in the country. The first four African languages are classified as Nguni while the next three are regarded as belonging to the Sotho family.

The constitution of SA guarantees equality for all languages in all sectors of the society including education. The use of African languages as a medium of instruction is strongly encouraged in the Foundation phase (up to Grade 3). However due to insufficiency of learning support materials in the black African languages as well as other reasons, Mathematics and Mathematical Literacy are taught in English as learners progress to higher grades. By the time African speaking learners come to be introduced to variables in Mathematics, their language of the instruction will be English. Looking at the word multiply, for instance, Farrugia (2009) argued that if English was to be promoted as the medium of instruction for mathematics, it had to be kept in mind that within the English register there were established ways of saying things. Hence, if it was chosen to promote an English mathematics register, this had to be accompanied by the awareness of the variation offered by related, but different, words (Farrugia 2009).This matter is further problematised by the fact that the African learners in SA have to start dealing with the variations between their home languages and English. The study by Planas (in press) showed how learners resorted to using their home language when referring to everyday life experiences and the official language when referring to their academic work.

The argument in this paper is that a detailed attention to language issues would help to better examine the critical opportunities for teaching and learning mathematics in multilingual classrooms in which participants –teachers and learners–are faced with in the representation of mathematics by means of ordinary languages with different grammars and ways of structuring meanings. This argument is made in two parts. First, it is claimed that mathematics teacher education needs to incorporate language issues for teachers to gain proficiency of both subject and pedagogy content. Second, using the multiplication of numbers, it is claimed that lack of attention to language issues may be an obstacle to the understanding of the meanings attached by learners in the operation of multiplied numbers. Although the data is situated in the context of multiplication, it has broader implications for mathematics teacher education and mathematics classrooms.

Conceptual Learning and Mathematics Teacher Education

Adler (2012) argues that it is taken for granted that the teaching activity is an important resource provided to the students for their learning. Like Windschitl (1999: 752), the researchers consider the scenario of active learners in collaboration with the teacher in the reorganization of knowledge to be important, where teaching should promote experiences that require learners to become active in the learning process, where "such experiences include problem-based learning, inquiry activities, dialogues with peers and teachers that encourage making sense of the subject matter, exposure to multiple sources of information, and opportunities for students to demonstrate their understanding in diverse ways". To achieve scenarios in this sense, teachers are to provide opportunities for conceptual learning, which in turn need to be grounded on their knowledge of potential differences in the representation and communication of the processes and products of learning.

Traditional mathematics classrooms, however, are characterised by teaching approaches that emphasise procedures, memorisation of algorithms and finding the one correct answer (Abramovich and Connell 2015) and when it comes to the teaching and learning of the operation of multiplication this kind of approach is often the one followed by teachers. Abramovich and Connell (2015) further argue that these approaches do not address reasoning, problem solving, and sense making in which discussions, debates, creative thinking and alternative ways of solving problems or alternative procedures for carrying out operations are encouraged. When it comes to the order of multiplied numbers Wiebe (1983) found that when pairs of multiplied numbers (15 and 1.25) are interchanged twice as many students got the task 1.25×15 right as compared to those who were correct on 15×1.25.

Teachers are therefore encouraged to embrace approaches that stimulate the construction of conceptual learning of multiplication. Ball (2003) states that being able to show how the multiplication algorithm is put together requires a kind of reasoned understanding beyond being able to use it fluently and accurately. Teachers have to know why the procedures work, that certain properties are true and particular relationships exist, as well as on what bases they do so. According to Fyfe et al. (2015) a lot of learners' time is spent on learning and practicing mathematical procedures.

Teachers have to design mathematically accurate explanations and tasks that are comprehensible and useful for the learning of multiplication (Lo et al. 2008). This is a huge challenge for some groups of teachers in many countries, particularly in South Africa, who have become mathematics teachers not because of the orientation in their initial training but due to the shortage of mathematics teachers in the country (Kaino et al. 2015). For these teachers, the ability to reason in the various mathematical domains is even more critical than for those teachers with an academic background of expertise in mathematics. For many of them, to use the diversity of meanings of the operation as the basis for their reasoning is not trivial, and consequently they do not model such experience of conceptual learning in their teaching activity in the classroom. Given the task of multiplying three by four, for instance, if they only interpret it as determining the one correct answer (Abramovich and Connell 2015), they are still far from other interpretations that are closer in meaning to the task of multiplying, for instance,the multiplication of three by an unknown (variable).

Language Issues and Mathematics Teacher Education

Howie (2004) reported on how South African learners tend to achieve higher scores in school mathematics when their proficiency in the language of instruction is higher. Speakers of English or Afrikaans at home tend to achieve higher scores in school mathematics while speakers of African languages are more likely to achieve lower scores (Howie 2004). The lack of resemblance between languages is described by Dawe (1983) as linguistic distance, and it has been used in contexts with different sets of languages (see, for instance, the discussion on Irish and English by NíRíordáin and O'Donoghue (2009), or that on Afrikaans, English and the African languages in South Africa by Gerber et al. (2005), who argue that there is a much smaller distance between Afrikaans and English in comparison to English and the African languages). The deficit position coming from the focus on difficulties due to the experience of linguistic distances has been contested by Planas and Setati-Phakeng (2014) in their interpretation of languages as resources. The idea of all languages as resources in the construction of meanings around school multiplication is often taken for granted.

What is not clear is the extent to which mathematics teachers and mathematics teacher educators are aware of the complexity of teaching mathematics to linguistically diverse learners, along with the complexity of moving from representing and communicating mathematical ideas in an ordinary language to another one. The researchers' interpretation of the awareness includes the attention to the linguistic structures in mathematical and ordinary languages as compared to the structures in the learners' home languages. Teachers and teacher educators may be aware of the linguistically diverse contexts in which the teaching activity takes place, but they may not be aware of the need to attend to language issues in a structured way in their programs. As stated by Essien (2010), despite the fact that most teachers teach mathematics in a language that is not their own, and to learners who are not English dominant, they do not address, in practice, the language issues in the design of their mathematics teaching.

The participants in this paper are faced with the double challenge of having to interpret the multiplication of numbers in ways that foster conceptual understanding and exploring meanings in this context that come from the use of different language structures. It is not a given that individuals would acquire the knowledge required in dealing with this challenge by the mere experience of being in a multilingual environment, but rather through forms of teacher education and professional development.

MATERIAL AND METHODS

Semi-structured interviews were conducted with African language-speakers who were teachers in multilingual mathematics classrooms. The paper reports on two participants who had more than ten years of teaching experience: Veli (male, 47 years old) and Dipou (female, 43 years old). At the time of the study, both teachers were registered at university doing part-time studies for a professional developmental course in mathematics teaching for the intermediate phase (teaching 10 to 12 year olds). They were teachers who at one or other time of their education had obtained apartheid era qualifications. Both teachers had been allocated to teach mathematics at their schools because of the shortage of mathematics teachers in the country. The apartheid-era Minister of Education, Verwoerd chose mathematics as the one subject not to be taught to the marginalised majority of South Africa under Bantu Education when he said in 1953: "There is no place for [the Bantu] in the European community above the level of certain forms of labour ... What is the use of teaching the Bantu child mathematics when it cannot use it in practice?" (Verwoerd 1960 as quoted in Pitoniak and Yeld 2013: 20). The legacy of this policy to the African teachers and learners of mathematics in the country is still recognisable in the mathematics classrooms.

After the democratic elections in South Africa in 1994, many teachers were re-deployed from one school to another and faced with the option of having to teach mathematics to remain employed. Most teachers would then register for this mathematics professional development course, which covered subject content and pedagogy. Veli was from the North West province where the SeTswana language is dominant, while Dipou was from Limpopo where the SePedi language is dominant. Despite the dominance of these languages in the provinces, the teachers are expected to teach mathematics in English from grade 4 onwards as per the education policy in SA.

The first author was the interviewer, and the researcher who took responsibility for collecting and recording data. At the beginning of the individual interviews, he posed to each teacher the task of interpreting a pair of multiplied numbers. From experience as a university teacher and a facilitator in courses of professional development in the country, he had observed the different interpretations of African speakers when compared to others (English and Afrikaans) in terms of multiplied numbers.

It is important to mention that the teachers were selected incidentally and they voluntarily availed themselves to be interviewed for the study. The teachers were interviewed using open-ended questions and they offered views but unfortunately no digital voice recording device but notes were used to capture their responses. The semi-structured questions sought to get the teachers' views relating to the interpretation and understanding of the notion of multiplication.

RESULTS

The Interviews

It is important to provide some background information relating to the interviews that were conducted. The concept of multiplication in the Sotho languages is described as '*atisa*' and this translates as *increase*. The Nguni version of the concept is '*phinda-phinda*' and it is translated as *repeat-repeat*. Of the two language groupings, the Nguni languages capture the 'repeated addition' notion of the concept of multiplication while there exists norepeat notion in the Sotho language mathematics register. This is despite the existence of the word '*boeletsa'* ('*repeat*') in the Sotho language grouping. In addition, the word '*atisa*' in the Sotho languages is more likely to be used in conjunction with '*ka'* (which translates as either *by*or *with*)to be '*atisaka*'. The word '*boeletsa*', however, is more likely to be used in conjunction with '*ga*'. When the word '*ga*' is used in the Sotho language, the notion of quantifying is implied as exemplified in the question '*gakae?*' ('*how many times?*'). For instance, something that is repeated '*five times* will be translated as '*gatlhano/hlano*' in the Sotho languages. When African speakers verbalise the reading of mathematics sentences involving the multiplication, say 2 x 3, they are, however, more prone to use the official language than their home languages (Planas, in press), reading the numbers in English and using the word *times* to read the multiplication sign.

The following excerpts captured the interesting discussions between the researcher and the two participants, who were teaching in provinces where the Sotho languages were dominant for the majority of their learners. They were asked to expand $3\times x$ and $x\times 3$.

The Case of Veli

The excerpt below corresponds with a moment in the interview with Veli, at the stage when the researcher was asking him to expand $x\times 3$:

Veli: It is $x+x+x$.*Ke di-x tse 3.* It is x's which are three in number. It is that simple.

Moshe: What about $3\times x$? [Writing it down on a paper]

Veli: It is three x. [Taking the pen from Moshe and writing $3x$down]. That's it.

Moshe: Do you know that others would represent this [pointing to $3\times x$] as $x+x+x$?

Veli: No.

Moshe: Yes.

Veli: Then they are confused.

The conversation above highlights how Veli resorted to using academic language as opposed to everyday language (Planas, in press) when responding to the question about $3\times x$, suggesting that he could not draw from his everyday language nor home language to describe this multiplication. The expectation of Veli responding with '3 + 3 + 3 + ….. (x-times)' for the expansion of $3\times x$ did not materialise. More importantly, however, it suggests the fact that African language-speakers interpret the multiplication numbers differently than ‘others’. Noting that Veli describes ‘others’ as ‘confused’ calls for a discussion and deliberation on the need to consider the meaning assigned to pairs of multiplied numbers by different groups of language speakers. The excerpt below involving another respondent gives credence to this proposition.

The Case of Dipou

The following exchange took place in another interview at the moment in which the same type of expression involving the commutative property of multiplication was being commented on:

Moshe: Is 3×4 representing $3+3+3+ 3$ or $4+ 4+4$?

Dipuo: It is $3+3+3+3$.

Moshe: What does $3\times x$ represent? [Writing it down]

Dipuo: *Di-3 tsemmh* {3’s which are *mmh*} [The sound *mmh* was mid-pitched]

In the extract above it is noticeable the consistency in which Dipuo answered the question by the researcher in line with how the African languages mention the object before the quantifier when they speak. Of even greater significance however, is the fact that not only is Dipuo identifying the *3* as the object but also replaces the *x* with *mmh*. The *mmh* is supposed to represent how many the objects (3’s) are indicated to be in number. The interjection *mmh?*, with a low pitch sound, is often used in the African languages when a listener wants the person saying something to repeat whatever it is the latter was saying (Planas, in press). With an exclamation, the interjection *(mmh!)*, with a high pitch sound, conveys an element of surprise or being perplexed on the part of the listener. The *mmh* uttered by Dipuo was neither a question nor an exclamation; it seemed somehow to be positioned between them. It was therefore uttered in such a way that Dipuo was caught between a person ready to provide a follow-up answer to the question being asked and a person being confused on what to say next.

DISCUSSION

More often than not, African speakers, as shown in the cases above, expand multiplied numbers such as 3×4 as $3+3+3+3$ instead of $4+4+4$. This difference in representation is not followed up for the African language-speaking learners because it is explained away through the commutative property. It is important to highlight that ignoring the attachment of relationships and meanings in the teaching and learning of multiplication whilst resorting to the use of processes such as memorising algorithms denies learners the opportunities of reasoning, problem solving, sense making, debates and creative thinking (Abramovich and Connell 2015). The argument here is that the understanding of the order of multiplied numbers should not be compromised by the use of the commutative property. Using the commutative property to explain away the differences between pairs of multiplied numbers that are interchanged also ignores findings such as that of Wiebe (1983). The current study reiterates the fact that individuals may attach different representations and/ or interpretationsin accordance to the order in which multiplied numbers are placed.

Table 1: Language representations of the relationship object-quantifier

Language		*Interpretation*
	Ordinary	*Mathematical*
English	a ***red*** wall	***Three*** boys
Afrikaans	ñ ***rooi***muur	***Drie*** seuns
IsiZulu	Idonga ***elibomvhu***	Abafana ***abathathu***
IsiSwati	Lubondza ***lolubovu***	Bafana ***labatsantfu***
IsiXhosa	Idhonga ***elibomvhu***	Amakhwenke ***amathathu***
IsiNdebele	Ibonda ***elibomvhu***	Abefana ***abathathu***
SePedi	Leboto le le***khibidu***	Bašimaneba ***bararo***
SeTswana	Lebota le le***hibidu***	basimaneba***bararo***
SeSotho	Lebota le le***gubedu***	Bashimaneba ***bararo***
XiTsonga	khumbi rho ***tshuka***	Bafana ***banaro***
TsiVhenda	Luvhondo ***lutshuku***	Vhathanga ***vhararu***

Moving from the premise that learners would tend to use procedures (Fyfe et al. 2015) for the purpose of getting the correct answer and not be involved in solving the problem (Abramovich and Connell 2015), when they give the answer of twelve as the product *3×4* or *4×3* they may not be questioned in terms of *how* they got to that answer. Multiplication is one activity – often through drilling–that learners can excel in without ever understanding.

A huge assumption is often made purporting that learning mathematics is fully achieved by learning its symbolic patterns. Research has however shown that learning the patterns that produce computational skill only, may lead to learners who may not progress beyond the understanding of howto use these patterns to solve problems (Abramovich and Connell 2015).

It is important, as argued above, to note that whilst the commutative property indicates that product *3×4* is the same as that of *4×3*, their representations are not the same. The representation of *3×4* is that of fours that are added and there are three of them, whereas *4×3* means that it is the threes that are added and they are four in number. The significance of this difference is crucial in the understanding of the concept of multiplication. However, the importance of this difference is explained away through the use of the commutative property. Whenever any two numbers are multiplied the first number assumes a descriptive nature, in particular in terms of quantifying, while the second number assumes the role of the object of the operation.

The descriptive role of the first number in multiplied numbers is important in mathematics. To demonstrate this point, take, for instance, if there are five apples to be counted, then *5× apples* (expanded form: *apple+apple+apple+ apple+apple*) will be a mathematical thereof. But it is difficult to bring an understandable representation to *apple×5* (expanded form: *5+5+ 5+... apple-times*). Until *apple* is quantified into a mathematical form no representation is possible for the expression *apple×5*. If another example of could be given, the description of a house as a *white* house–compared to *one* house–has no relevance in terms of a mathematical description if the *white* only refers to colour. In order for this description *white* to have mathematical relevance, for instance, it could be viewed in terms of how much of the house is white, say, 50 percent or 30 percent *white*, etc. or, if there existed another (or more) house(s) that were also white, to describe them in terms of *how many* white houses were there. More importantly, however, to argue that *3×4* is the same as *4×3* simply because they both result in twelve is also to ignore the innumerable other pairs of numbers that result in twelve when operated, not only by multiplication but by other operations as well.

It is against the background of looking at the descriptive role played by one of the numbers when two numbers are multiplied that the researchers suggest that the understanding of mathematics by African speakers in South Africa is compromised by resorting to explaining the differences away–by alternating the positions of numbers – through the use of the commutative property. One cannot help it but highlight how algebraic terms are conventionally presented in mathematics in exactly the way the non-African languages – in the South African context, English and Afrikaans– are communicated.

In the English and Afrikaans languages the description of any object is always put in front of the described noun in the form of an adjective, for example 'a red wall' or *'ñ rooimuur'*, and 'three boys' or *'drieseuns'*. It therefore raises no eyebrows for these language speakers to represent $3 \times x$ as $3x$ in mathematics. The results of the current study, however, showed that $x \times 3$ or $x3$ elicited an understanding for African speaking learners that mapped their way of communication. In the African languages of South Africa the description of any object at most times follows the object that is being described. As shown in Table 1 the description of the object – in ***bold italics***– in African languages differs with that of the English and the Afrikaans languages in which the description precedes the described object.

The following statements are used to depict the non-African interpretation (which is also the mathematical convention) of the product of 3 and *x*:

(1) $3 \times x$which means $x+x+x$

(2) $x \times 3$ which means 3+3+3+3+… (*x* times)

The representation of statement (1) is clear, while the representation of statement (2) leaves a degree of uncertainty in terms of the description of the mentioned objects (3's) until *x* takes the form a number or is mathematized. Considering the importance of the mathematical description of objects that is sought, this uncertainty will bring with it a lesser understanding of statement (2). Rather than saying that it is an accepted norm to depict $x\times 3$ as $3x$ and not as $x3$, it is better to say that it is more advantageous to depict $x\times 3$ as $3x$ as it provides an improved understanding. The learning of variables in mathematics comes much later long after the commutative property has been learnt by learners and therefore an African learner that has the understanding of the multiplication of numbers in a way that is consistent with how objects are described in the African language will interpret $3\times x$ as $3+3+3+\ldots$(x times). This should bring with it the element of uncertainty already described.

Implications

The researchers argue that the understanding of multiplication needs to be treated with the caution it deserves when it comes to its representation to African learners. It is firstly important for the representation of pairs of numbers not to be explained away by teachers, at least not only, through the commutative property. Secondly and even more significantly is to ensure that the representations of pairs of multiplied numbers are understood by African speakers in the same way it is understood by speakers of either English or Afrikaans. It is encouraging to realise that it is possible for the African languages to describe objects in mathematics in a way that is consistent with the way mathematics is written. What is of interest is to note that the African languages use the way of description that is consistent with the one of mathematics when the numbers that are involved are either huge or fractions (see Table 2).

Describing objects using numbers in a manner that is shown in Table 2 is not used consistently to enhance improved understanding of multiplied numbers for African learners. Awareness on how African languages express themselves in the description of objects using numbers is necessary so that an alignment with the one used in mathematics can be made. Table 3 points to alternative ways of expression for African languages which are not different to that of English and the mathematics one.

The alternative way of expressing quantified objects in the African languages not only offers learners from these backgrounds with the understanding that is consistent with how they are represented in mathematics and languages of European origin in the country but ensures that these learners' understanding of algebraic terms or expressions is not compromised how they are interpreted in their languages.

CONCLUSION

Putting forward algorithms or rules in the name of ´doing´ mathematics compromises on the most important aspect of the understanding of the subject of mathematics. Careful use of rules and algorithms should be accompanied by appropriate understanding of the concepts and definitions that are used. For African language-speakers in South Africa, this matter has not been given the suitable attention that it deserves. The pursuance of having to engage learners in activities that may be classified as learners 'do-

Table 2: Language representations of the relationship object-"huge" quantifier

Language	*Interpretation*	
English	A ***thousand*** people	***Half*** of the workers
Afrikaans	ñ ***duisend*** mense	ñ ***helfde*** van die werkers
IsiZulu	***Inkulungwane*** ya bantu	***Inhlangothi*** yabasebenzi
SeTswana	***Sekete*** sabatho	***Halofo*** yabadiri

Table 3: Alternative language representations of the relationship object-quantifier

Language	*Normal Usage*	*Alternative Way*
English	*Eleven* players	*Eleven* players
IsiZulu	Abadlali aba ***ishumenanye***	*Ishumenanye* la badlali
SePedi	**Baralokiba *lesome-tee***	*Lesome-tee* la baraloki
SeTswana	**Batshamekiba *lesome-nngwe***	*Lesome-nngwe*la batshameki
SeSotho	Dibapaditse ***leshome-nngwe***	*Leshome-nngwe* la dibapadi
XiTsonga	Vhanhlagivha ***khomeyingwe***	***Khomeyingwe*** yavhanhlagi

ing' mathematics implores us to take matters such as the ones raised in this paper as critical for the understanding of mathematics.

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